Testing Positivity at Muon Colliders

Jiayin Gu

JGU Mainz
(⇒ Fudan U. this Fall)

APS April Meeting 2021, Y07 Muon Collider Symposium IV
April 20/21, 2021

[arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang

Introduction

- **Hope for the best!**
  - Maybe we will directly discover new physics! (Forget about EFT...)
- **Prepare for the worst!**
  - No discovery? We can still learn a lot!
- Precision measurements (Higgs, EW, top, ...)
  - The Standard Model Effective Field Theory (SMEFT) is a good framework.
  - **Energy** *vs.* **Precision ...**

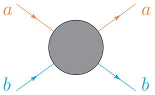
Introduction

- **Hope for the best!**
  - Maybe we will directly discover new physics! (Forget about EFT...)
- **Prepare for the worst!**
  - No discovery? We can still learn a lot!
- Precision measurements (Higgs, EW, top, ...)
  - The Standard Model Effective Field Theory (SMEFT) is a good framework.
  - **Energy** *vs.* **Precision ...**
- **Can all EFTs be UV completed?**
- Dispersion relations of forward elastic amplitudes suggest that certain operator coefficients can only be positive.
  - Assuming the UV physics is consistent with the fundamental principles of QFT (analyticity, locality, unitarity, Lorentz invariance).
  - [hep-th/0602178] Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, + many papers...

Introduction

- **These positivity bounds only exist for certain Dimension-8 (or higher) operators!**

$$\frac{d^2}{ds^2}\mathcal{A}(ab \to ab)_{t\to 0}\big|_{s=0} \geq 0\,.$$

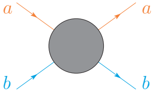


- By measuring these dim-8 operator coefficients, we can test whether the underlying new physics is consistent with the fundamental principles of QFT.

Introduction

- **These positivity bounds only exist for certain Dimension-8 (or higher) operators!**

$$\frac{d^2}{ds^2}\mathcal{A}(ab \to ab)_{t\to 0}\big|_{s=0} \geq 0\,.$$

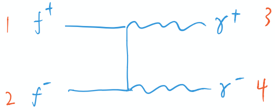


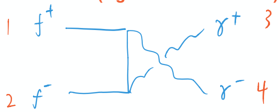
- By measuring these dim-8 operator coefficients, we can test whether the underlying new physics is consistent with the fundamental principles of QFT.

- **Can we do it?**
  - It's difficult to probe dim-8 operators!
  - **Energy AND Precision!** (A linear collider or a muon collider)
  - Or find some special process where dim-8 gives the leading new physics contribution? (main focus of this talk)

The diphoton channel [arXiv:2011.03055] JG, Lian-Tao Wang, Cen Zhang

(signs denote helicities)





- $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$), SM, non-resonant.
    - Tree level SM: the only helicity configuration is $\mathcal{A}(f^+f^-\gamma^+\gamma^-)$.
- Leading order contribution: **dimension-8 contact interaction**. ($f^+f^- \to \bar{e}_L e_L$ or $e_R\bar{e}_R$)

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\mathrm{SM+d8}} = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 .$$

- Operators: Also contributes to $ZZ/Z\gamma$ final states with opposite helicities.

  [1806.09640] Bellazzini, Riva, see also d8 basis in

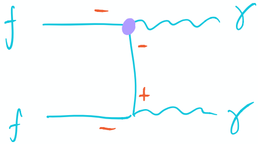
  [2005.00008] Shu et al., [2005.00059] Murphy

$$a_L = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{\ell B} - \cos\theta_W \sin\theta_W\, c^{(8)}_{\ell BW} + \sin^2\theta_W\, c^{(8)}_{\ell W}\right) ,$$
$$a_R = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{eB} + \sin^2\theta_W\, c^{(8)}_{eW}\right) ,$$

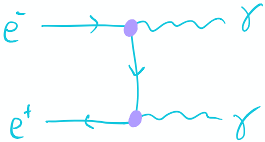
$$\begin{aligned}
\mathcal{O}^{(8)}_{\ell B} &= -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho} ,\\
\mathcal{O}^{(8)}_{eB} &= -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \mathrm{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho} ,\\
\mathcal{O}^{(8)}_{\ell W} &= -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})W^a_{\mu\nu}W^{a\,\mu}{}_{\rho} ,\\
\mathcal{O}^{(8)}_{eW} &= -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \mathrm{h.c.})W^a_{\mu\nu}W^{a\,\mu}{}_{\rho} ,\\
\mathcal{O}^{(8)}_{\ell BW} &= -\frac{1}{4}(i\bar{\ell}_L\sigma^a\gamma^{\{\rho}D^{\nu\}}\ell_L + \mathrm{h.c.})B_{\mu\nu}W^{a\,\mu}{}_{\rho} ,
\end{aligned}$$

All other contributions are either vanishing or suppressed!

- **The only tree-level d6 contribution are from dipole operators and have different fermion helicities.**

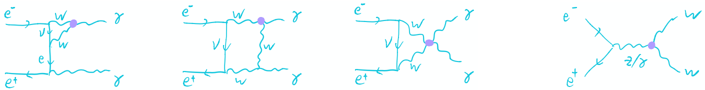


- **SM×d6 at tree level:** no interference.
- **d6^2:** Dipole operators are very well constrained by $g-2$ and EDM measurements.

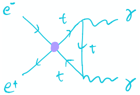


All other contributions are either vanishing or suppressed!

- **SM×d6 at 1-loop:** are either very-well constrained by other measurements with tree-level contributions, or forbidden by selection rules.
  - O_{3W} is very well constrained by $e^+e^- \to WW$ measurements.



  - Other contributions are constrained by Z-pole measurements or suppressed by the small y_e.
  - Contribution from the *eett* 4f operator is forbidden by angular momentum selection rules. ([2001.04481] Shu et al.)



$= 0$ (for e^+e^- with opposite helicities)

- **other d8:** They have different helicities and do not interfere with SM.

Positivity bounds

- Leading BSM contribution:

$$\mathcal{A}(\bar{e}_L e_L \gamma^+ \gamma^-)_{\rm d8} = \frac{a_L}{v^4}[13][23]\langle 24\rangle^2\,, \qquad \mathcal{A}(e_R \bar{e}_R \gamma^+ \gamma^-)_{\rm d8} = \frac{a_R}{v^4}[13][23]\langle 24\rangle^2\,.$$

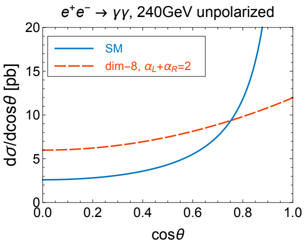
- Positivity bounds are obtained from the forward elastic amplitude $e\gamma \to e\gamma$:

$$\frac{d^2}{ds^2}\mathcal{A}(e\gamma \to e\gamma)|_{t\to 0} \geq 0\,,$$

- which implies

$$a_L \geq 0\,, \quad a_R \geq 0\,.$$

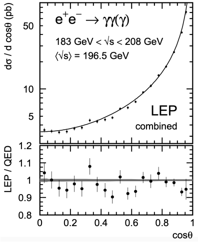
The diphoton cross section



- Differential cross section (The production polar angle θ is "folded" since the photon polarizations are not measured.)

$$\frac{d\sigma(e^+e^-\to\gamma\gamma)}{d\cos\theta} = \frac{(1-P_{e^-})(1+P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2}+a_L\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right) + \frac{(1+P_{e^-})(1-P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2}+a_R\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right),$$

1302.3415

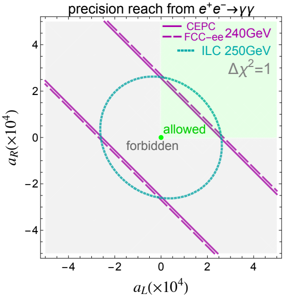


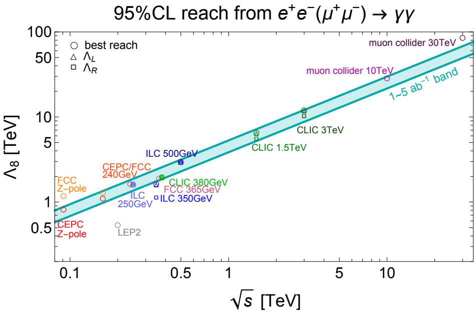
- Positivity bounds: $a_L \geq 0$, $a_R \geq 0$.
- **Positivity bound directly on the cross section!**

$$\sigma(e^+e^-\to\gamma\gamma) \geq \sigma_{\rm SM}(e^+e^-\to\gamma\gamma)\,.$$

- The LEP measurement was $\sim 1.5\sigma$ below the SM prediction.

Future projections





- χ^2 fit to the binned distribution
  - Statistics only, 19 bins in $\cos\theta \subset [0, 0.95]$.
  - Agrees reasonably well with LEP result ($\lesssim 10\%$ in the reach on Λ).
- Is beam polarization useful? **Yes and no!**
  - One could measure σ_L and σ_R simultaneously.
- High energy still wins!

$$\frac{\Lambda_2}{\Lambda_1} = \left(\frac{E_2}{E_1}\right)^{\frac{3}{4}} \left(\frac{L_2}{L_1}\right)^{\frac{1}{8}} .$$

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy

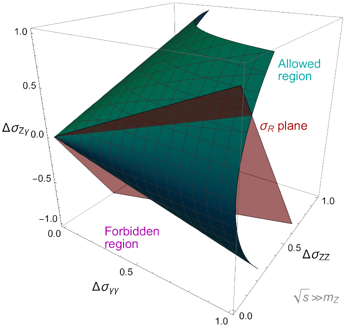
- $Z\gamma$, ZZ processes are more complicated due to the massive Z.
  - Other helicity states contribute in both SM and BSM (*e.g.* nTGCs).
  - In the high energy limit, $\mathcal{A}(f^+ f^- V^+ V^-)$ dominates in SM.

- In the $\sqrt{s} \gg m_Z$ limit,

$$\sigma(e^+e^- \to ZZ) \geq \sigma_{\rm SM}(e^+e^- \to ZZ)\,.$$

- Consider the elastic amplitude of $eV \to eV$,
  - V is an arbitrary mixing state of γ and Z,
  - scan over the mixing angle to obtain the strongest bound ($\Delta\sigma \equiv \sigma - \sigma_{\rm SM}$),

$$(\Delta\sigma_{Z\gamma})^2 \leq 4\Delta\sigma_{\gamma\gamma}\Delta\sigma_{ZZ}\,.$$



- σ_Rs only occupy a plane in the 3d parameter space.
  - 3 operators with ℓ_L, 2 operators with e_R.

$e^+e^- \to e^+e^-$ [2009.02212] Fuks, Liu, Zhang, Zhou

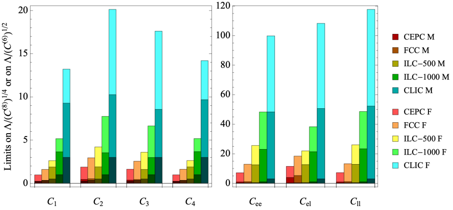
- Many operators, many positivity bounds...

$$O_{ee} = (\bar{e}\gamma^\mu e)\,(\bar{e}\gamma_\mu e)\,,\quad O_{el} = (\bar{e}\gamma^\mu e)\,(\bar{l}\gamma_\mu l)\,,\quad O_{ll} = (\bar{l}\gamma^\mu l)\,(\bar{l}\gamma_\mu l)\,,$$

$$\begin{aligned} O_1 &= \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\,, \\ O_2 &= \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\,, \\ O_3 &= D^\alpha(\bar{e}l)\;D_\alpha(\bar{l}e), \\ O_4 &= \partial^\alpha(\bar{l}\gamma^\mu l)\;\partial_\alpha(\bar{l}\gamma_\mu l)\,, \\ O_5 &= D^\alpha(\bar{l}\gamma^\mu\tau^I l)\;D_\alpha(\bar{l}\gamma_\mu\tau^I l)\,, \end{aligned}$$

$$\begin{aligned} &C_1 \le 0, \\ &C_4 + C_5 \le 0, \\ &C_5 \le 0, \\ &C_3 \ge 0, \\ &2\sqrt{C_1(C_4+C_5)} \ge C_2, \\ &2\sqrt{C_1(C_4+C_5)} \ge -(C_2+C_3). \end{aligned}$$

- **Multiple runs with different energies** & beam polarizations are very useful! Angular distributions also help.



Conclusion

- $e^+e^- \to \gamma\gamma$ or $\mu^+\mu^- \to \gamma\gamma$ offers a unique opportunity to directly probe dimension-8 operators and their positivity bounds.
- We can do it at a lepton collider with $\sqrt{s} \sim 240$ GeV.
- A muon collider can do **much better AND much more** because it has **energy AND precision!**
  - dilepton, diboson, VBS, ...
  - basically uncharted territory...

Conclusion

- $e^+e^- \to \gamma\gamma$ or $\mu^+\mu^- \to \gamma\gamma$ offers a unique opportunity to directly probe dimension-8 operators and their positivity bounds.
- We can do it at a lepton collider with $\sqrt{s} \sim 240$ GeV.
- A muon collider can do **much better AND much more** because it has **energy AND precision!**
  - dilepton, diboson, VBS, ...
  - basically uncharted territory...

"When you have excluded the impossible, whatever remains, however improbable, must be the truth."

– Sherlock Holmes

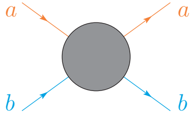
backup slides

Dispersion relations

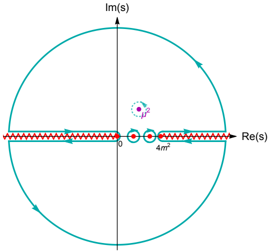
- Consider a forward ($t \to 0$) elastic amplitude ($s + t + u = 4m^2$)

$$\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s - \mu^2)^n ,$$

$$c_n = \frac{1}{2\pi i} \oint_{s=\mu^2} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s - \mu^2)^{n+1}} ,$$



- Applying the fundamental principles of QFT
  - Analyticity (Cauchy's theorem applies)
  - Locality (poles from tree-level factorization, branch cuts from loops, Froissart Bound)
  - Unitarity (Optical theorem, $\mathrm{Im}\mathcal{A} \sim \sigma_{\mathrm{tot}}$)
  - Lorentz invariance (Crossing symmetry)



- Dispersion relation tells us that

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma^{ab}_{\mathrm{tot}}}{(s - \mu^2)^{n+1}} + (-1)^n \frac{\sigma^{a\bar{b}}_{\mathrm{tot}}}{(s - 4m^2 + \mu^2)^{n+1}} \right) + c_n^{\infty} ,$$

Sum rules and positivity bounds

- Sum rule:

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s\sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma^{ab}_{\rm tot}}{(s-\mu^2)^{n+1}} + (-1)^n \frac{\sigma^{a\bar{b}}_{\rm tot}}{(s-4m^2+\mu^2)^{n+1}} \right) + c_n^{\infty} \,,$$

  - Froissart bound: $\mathcal{A} < \mathrm{const} \cdot s \log^2 s \quad \Rightarrow \quad c_n^{\infty} = 0$ for $n > 1$.
  - For even n, the two terms with cross sections are both positive, so $c_n > 0$.

- Consider the limit $m^2 \ll \mu^2 \ll \Lambda^2$ (massless SMEFT).

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \cdots \,.$$

$$\mathcal{A}_4 = g_{[0]} \mathcal{A}_4^{[0]} + g_{[-2]} \mathcal{A}_4^{[2]} + g_{[-4]} \mathcal{A}_4^{[4]} + \ldots.$$

- $c_{n=1}$ $\Leftrightarrow$ dimension-6 (no positivity bounds, boundary can be nonzero),
  $c_{n=2}$ $\Leftrightarrow$ dimension-8 (or d6^2) (has positivity bounds),
  ...

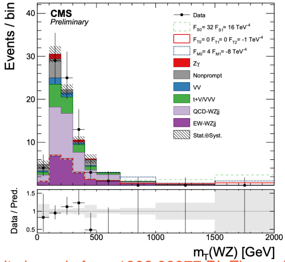
- See *e.g.* [2011.00037] Bellazzini, Miró, Rattazzi, Riembau, Riva, [2012.15849] Arkani-Hamed, Huang, Huang for more general positivity bounds also for non-forward amplitudes.

Probing positivity bounds on dimension-8 operators

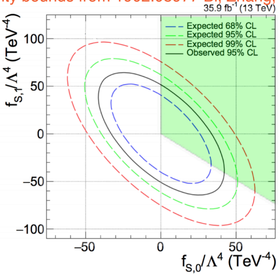
- The dimension-8 contribution has a large energy enhancement ($\sim E^4/\Lambda^4$)!
- It is difficult for LHC to probe these bounds.
  - Low statistics in the high energy bins.
  - Example: Vector boson scattering.
  - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- Can we separate the dim-8 and dim-6 effects?
  - Precision measurements at several different $\sqrt{s}$?

    (A **very** high energy lepton collider?)
  - Or find some special process where dim-8 gives the leading new physics contribution?

CMS-PAS-SMP-18-001



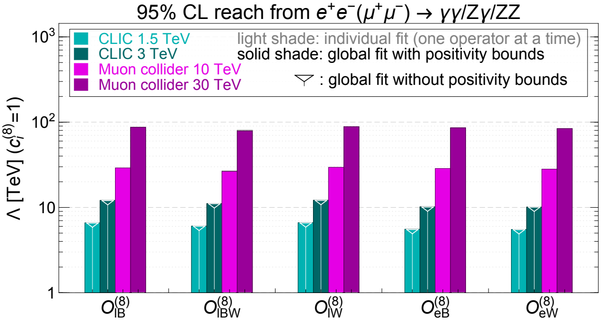
positivity bounds from 1902.08977 Bi, Zhang, Zhou



Collider scenarios

$\int \mathcal{L}dt$ [ab^{-1}]				
unpolarized	91 GeV	161 GeV	240 GeV	365 GeV
CEPC	8	2.6	5.6	
FCC-ee	150	10	5	1.5
ILC	250 GeV	350 GeV	500 GeV	
$(-0.8, +0.3)$	0.9	0.135	1.6	
$(+0.8, -0.3)$	0.9	0.045	1.6	
$(\pm 0.8, \pm 0.3)$	0.1	0.01	0.4	
CLIC	380 GeV	1.5 TeV	3 TeV	
$(-0.8, 0)$	0.5	2	4	
$(+0.8, 0)$	0.5	0.5	1	
muon collider	10 TeV	30 TeV		
unpolarized	10	90		

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy



- Global fit of the $\gamma\gamma/Z\gamma/ZZ$ processes in the high energy limit.
- No beam polarizations $\Rightarrow$ flat directions.
- Flat directions are lifted once the positivity bounds are imposed!

What if positivity bound is violated?

- Statistical fluctuation, systematic error, ...
  - Even 5σ can go away in the diphoton channel.
- EFT is not valid?
  - An s-channel light ($m \lesssim \sqrt{s}$) spin-2 particle?
  - Very well probed by resonance searches $e^+e^- \to X\gamma/XZ$, $X \to \gamma\gamma/e^+e^-$. (see *e.g.* ILC 750 GeV study [1607.03829])
  - By measuring $e^+e^- \to \gamma\gamma$ at several energies (*e.g.* Z-pole and 240 GeV) we can check whether the deviation comes from d8 operators ($\sim s^2$) or something else.
- Can QFT really break down at the TeV scale?
  - Example from history: Nobody expected classical physics to break down, nobody expected parity to be violated, ...

What if positivity bound is violated?

- Statistical fluctuation, systematic error, ...
  - Even 5σ can go away in the diphoton channel.
- EFT is not valid?
  - An *s*-channel light ($m \lesssim \sqrt{s}$) spin-2 particle?
  - Very well probed by resonance searches $e^+e^- \to X\gamma/XZ$, $X \to \gamma\gamma/e^+e^-$. (see *e.g.* ILC 750 GeV study [1607.03829])
  - By measuring $e^+e^- \to \gamma\gamma$ at several energies (*e.g.* Z-pole and 240 GeV) we can check whether the deviation comes from d8 operators ($\sim s^2$) or something else.
- Can QFT really break down at the TeV scale?
  - Example from history: Nobody expected classical physics to break down, nobody expected parity to be violated, ...

- **It's important to do the experiment!**

Sum rules for dimension-6 operators [arXiv:2008.07551] JG, L.-T. Wang

- Use helicity amplitudes to classify the sum rules.

elastic 4-point amplitudes $\mathcal{A}(1\,2 \;\to\; 3_{=1}\,4_{=2})$	spinor form of $\mathcal{A}_4^{[2]}$ (d6 operators)	spinor form of $\mathcal{A}_4^{[4]}$ (d8 or d6^2)
$\phi_1\phi_2\phi_1^*\phi_2^*$	s_{ij}	$s_{ij} \times s_{kl}$
$\psi^-\phi\psi^+\phi^*$	$\langle 12\rangle[23]$	$\langle 12\rangle[23] \times s_{ij}$
$\psi_1^-\psi_2^-\psi_1^+\psi_2^+$	$\langle 12\rangle[34]$	$\langle 12\rangle[34] \times s_{ij}$
$V^-\phi V^+\phi^*$	✗	$\langle 12\rangle^2[23]^2$
$V^-\psi^-V^+\psi^+$	✗	$\langle 12\rangle^2[23][34]$
$V_1^-V_2^-V_1^+V_2^+$	✗	$\langle 12\rangle^2[34]^2\,,\;\langle 12\rangle^2[34]^2\frac{t-u}{s}$

- Tree level dimension-6: only scalar-scalar, scalar-fermion and fermion-fermion amplitudes!

- Forward limit:

$$\tilde{\mathcal{A}}_4^{[2]} \equiv \mathcal{A}_4^{[2]}|_{t\to 0} \propto s\,, \qquad \tilde{\mathcal{A}}_4^{[4]} \equiv \mathcal{A}_4^{[4]}|_{t\to 0} \propto s^2\,.$$

Sum rules

$\mathcal{O}_H = \frac{1}{2}(\partial_\mu \|H\|^2)^2$	$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_{H\ell} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{\ell}_L \gamma^\mu \ell_L$ $\mathcal{O}'_{H\ell} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{\ell}_L \sigma^a \gamma^\mu \ell_L$	$\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R \gamma^\mu e_R$
$\mathcal{O}_{Hq} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{q}_L \gamma^\mu q_L$ $\mathcal{O}'_{Hq} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}_L \sigma^a \gamma^\mu q_L$	$\mathcal{O}_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{u}_R \gamma^\mu u_R$ $\mathcal{O}_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{d}_R \gamma^\mu d_R$

- scalar-scalar

$$\frac{c_H + 3c_T}{\Lambda^2} = \left.\frac{d\tilde{A}_{\phi^+\phi^-}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{\phi^+\phi^-} - \sigma_{\rm tot}^{\phi^+\phi^+}\right) + c_\infty\,,$$

$$-\frac{2c_T}{\Lambda^2} = \left.\frac{d\tilde{A}_{\phi^+\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{\phi^+\phi^0} - \sigma_{\rm tot}^{\phi^+\phi^{0*}}\right) + c_\infty\,,$$

- scalar-fermion

  (also the same for leptons)

$$\frac{2(c_{Hq} - c'_{Hq})}{\Lambda^2} = \left.\frac{d\tilde{A}_{u_L\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{u_L\phi^0} - \sigma_{\rm tot}^{u_L\phi^{0*}}\right) + c_\infty\,,$$

$$\frac{2c_{Hu}}{\Lambda^2} = \left.\frac{d\tilde{A}_{u_R\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{u_R\phi^0} - \sigma_{\rm tot}^{u_R\phi^{0*}}\right) + c_\infty\,,$$

$$\frac{2(c_{Hq} + c'_{Hq})}{\Lambda^2} = \left.\frac{d\tilde{A}_{d_L\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{d_L\phi^0} - \sigma_{\rm tot}^{d_L\phi^{0*}}\right) + c_\infty\,,$$

$$\frac{2c_{Hd}}{\Lambda^2} = \left.\frac{d\tilde{A}_{d_R\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{d_R\phi^0} - \sigma_{\rm tot}^{d_R\phi^{0*}}\right) + c_\infty\,,$$

- fermion-fermion

  only showing $\frac{c_{ee}}{\Lambda^2}(\overline{e_R}\gamma_\mu e_R)(\overline{e_R}\gamma^\mu e_R)$,

  20 in total for 1 generation

$$-\frac{2c_{ee}}{\Lambda^2} = \left.\frac{d\tilde{A}_{e_R\overline{e_R}}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{e_R\overline{e_R}} - \sigma_{\rm tot}^{e_R e_R}\right) + c_\infty\,.$$

Example: $Zb\bar{b}$ Custodial symmetry

- How the $Zb_L\bar{b}_L$ couplings is related to heavy quarks.

$$\frac{4\,\delta g_{Lb}}{v^2} = -\frac{2(c_{Hq}+c'_{Hq})}{\Lambda^2} = \left.\frac{d\tilde{A}_{t_L\,\phi-}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{t_L\,\phi^-\to F^{-\frac{1}{3}}} - \sigma^{t_L\,\phi^+\to F^{\frac{5}{3}}}\right) + c_\infty\,,$$

- We can impose some symmetry to ensure the cancellation of the two cross section terms. ($Zb\bar{b}$ custodial symmetry, [hep-ph/0605341] Agashe et al.)

Sum rule on δg_{Lb}